Printed Page:- 05 **Subject Code:- BMIAS0203**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: II - THEORY EXAMINATION (2023- 2024)

Subject: Engineering Mathematics-II

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. 1

Degree and order of the differential equation $\sqrt{\left(\frac{dy}{dx}\right)^2 + 3y} = \frac{d^2y}{dx^2}$ is (CO1)

(a) Ord = 2, Deg = 2

(b) Ord = 2, Deg = 1

(c) Ord = 1 , Deg = 1

(d) Ord = 1 , Degree = 2

1-b. The P. I of the differential equation $(D^2 + 4)y = Cos2x$ is (CO1) 1

(a) $\frac{x}{4}\cos 2x$

(b) $\frac{x}{4}\sin 2x$

(c) $x\cos 2x$

(d) None of these

1-c. The Fourier coefficients a_n for the function $f(x) = x,\ 0 < x < 2\pi$, is (CO2) 1

(a) 1

(b) π

(c) $-\pi$

(d) 0

1-d. The series $\sum_{n=1}^{\infty} u_n$ of positive terms is divergent if $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right)$ is (CO2) 1

(a) < 1

(b) > 1

(c) $= 1$

(d) none of these

1-e. Inverse Laplace of the function $f(s) = \frac{1}{2s+3}$ is (CO3) 1

(a) $-\frac{1}{2}e^{-3t/2}$

(b) $-\frac{1}{2}e^{3t/2}$

(c) $\frac{1}{2}e^{-3t/2}$

(d) None of these

1-f. Laplace transform of $e^{-3t}u(t-2)$ is (CO3) 1

(a) $\frac{e^{-2(s+3)}}{s-3}$

(b) $\frac{e^{-2(s+3)}}{s+3}$

(c) $\frac{e^{-2(s+3)}}{s^2+3}$

(d) $\frac{e^{-2(s+3)}}{s^2-3}$

1-g. If $\vec{V}$ is linear velocity and $\vec{W}$ is angular velocity, then which of the statements is true? (CO4) 1

(a) $\vec{W} = \text{curl}\,\vec{V}$

(b) $\vec{W} = 2\text{curl}\,\vec{V}$

(c) $\vec{W} = \frac{1}{2}\text{curl}\,\vec{V}$

(d) $\vec{W} = \text{div}\,\vec{V}$

1-h. Green's Theorem state that (CO4) 1

(a) $\int_c (M\,dx - N\,dy) = \int\int_R (\partial N/\partial x - \partial M/\partial y)\,dx\,dy$

(b) $\int_c (M\,dx + N\,dy) = \int\int_R (\partial N/\partial x - \partial M/\partial y)\,dx\,dy$

(c) $\int_c (M\,dx + N\,dy) = \int\int_R (\partial N/\partial y - \partial M/\partial x)\,dx\,dy$

(d) $\int_c (M\,dx + N\,dy) = \int\int_R (\partial N/\partial y + \partial M/\partial x)\,dx\,dy$

1-i. A sum of money at simple interest amount to Rs 1045 in 5 years and to Rs 1111 in 6 years.The sum is (CO5) 1

(a) Rs 945
(b) Rs 715
(c) Rs 845
(d) Rs 775

1-j. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl? (CO5) 1

(a) Brother
(b) Nephew
(c) Uncle
(d) Son-in-law

2. Attempt all parts:-

2.a. Find the P.I of the differential equation $(4D^2+4D-3)y=e^{2x}$.(CO1) 2

2.b. Find the Fourier coefficient a_0 in $(0, 2)$ for $f(x)=\begin{cases} x, & 0<x<1 \\ 0, & 1<x<2 \end{cases}$ (CO2) 2

2.c. Find Laplace transform of the function $\sin t.\ u(t-\pi)$. (CO 3) 2

2.d. Show that vector $\vec{V}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x-2z)\hat{k}$, is solenoidal. (CO4) 2

2.e. Pointing to a photograph, Rahul said, "She is the mother of the wife of my brother's father". How is the lady in the photograph related to Rahul? (CO5) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Solve the differential equation: $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\sin(\log x^2)$.(CO1) 6

3-b. Solve : $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$, given that $x=2$ and $y=0$ when $t=0$. (CO1) 6

3-c. Obtain the Fourier series for $f(x)=\left(\frac{\pi-x}{2}\right),\ 0<x<2$. (CO2) 6

3-d. Test the series: $\frac{14}{1^3}+\frac{24}{2^3}+\frac{34}{3^3}+\ldots\ldots\ldots+\frac{10n+4}{n^3}+\ldots\ldots\ldots$ (CO2) 6

3.e. Evaluate the value of the integral $\int_0^\infty e^{-2t}\ \sin^3 t\ dt$. (CO 3) 6

3.f. Find the directional derivative of $f=x^2-y^2+2z^2$ at the point $P\ (1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$. (CO4) 6

3.g. Sneha's mother's age is five years more than twice the age of Sneha. When Sneha was born, her brother Rahul was four years old and her father two years older than her mother. If the average age of her mother and father is 56 years. Find the ratio 6

of age of Rahul to that of Sneha. (CO5)

SECTION-C 50

4. Answer any one of the following:-

4-a. Solve the following differential equation by changing the independent variable: 10

$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$$.(CO1)

4-b. 10

Solve the differential equation: $\frac{d^2y}{dx^2} + a^2y = \tan ax$.(CO1)

5. Answer any one of the following:-

5-a. 10

Obtain the Fourier series for the function $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$. (CO2)

Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots\ldots\ldots = \frac{1}{2}$

5-b. Test the convergence of the series 10

$$x + \frac{1}{2}.\frac{x^3}{3} + \frac{1.3}{2.4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + \ldots\ldots\ldots$$ (CO2)

6. Answer any one of the following:-

6-a. Using Convolution Theorem evaluate 10

$$L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$$. (CO3)

6-b. Solve the following differential equation by using Laplace transform 10

$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$, *Given that* $x = 2, \frac{dx}{dt} = -1$ *at* $t = 0$. (CO3)

7. Answer any one of the following:-

7-a. 10

Verify Divergence theorem for $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by the planes
x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. (CO4)

7-b. Verify Stoke's Theorem for function $\vec{F} = xz\,\hat{i} - y\,\hat{j} + x^2y\,\hat{k}$, where the surface S is the surface of the region bounded by $x = 0, y = 0, z = 0, 2x + y + 2z = 8$ which is not included on xz-plane . (CO-4) 10

8. Answer any one of the following:-

8-a. (i) Two vessels contain milk and water in ratio 3:2 and 7:3. Find the ratio in which the contents of the two vessels have to be mixed to get a new mixture in which the ratio of milk and water is 2:1. 10
(ii) Ashish has to go to his coaching class 5 days in a week. He walks to the Institute all by himself. Starting from his house, he starts moving East and walks 90 m. He then turned right and walked 20 m. He then took a right turn and walked for 30 m. From there, Ashish moved 100 m to the north and reached his Coaching Institute. How far is his house from the coaching centre? (CO5)

8-b. (i) A certain sum is to be divided between A and B so that after 5 years the amount received by A is equal to the amount received by B after 7 years. The rate of interest is 10%, interest compounded annually. Find the ratio of amounts invested by them . 10

(ii) Rs.1060 is divided into three parts in such a way that half of the first part, one-third of the second part and one-fifth of the third part are in the ratio 4 : 5 : 6. Find the second part . (CO5)